

# *On Classical Solutions of Vlasov-Poisson Fokker-Planck Systems*

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ABSTRACT. The theory developed for collisionless Vlasov-Poisson models is extended to include Vlasov dynamics for models with collision effects. For such models, the motion of a particle is resolved into the influence of self-consistent forces and dynamical friction forces, due to the interaction with the surrounding medium or fluid, and into a fluctuating part characteristic of Brownian motion. We derive global existence of classical solutions for arbitrary data when the spatial, or momentum, dimension is less than, or equal to, two, but local existence for arbitrary data in all other dimensions. This is effected, as in the collisionless cases, by obtaining a-priori estimates on the charge or mass density. The key to developing our theory lies in a procedure for constructing a fundamental solution to a class of degenerate, linear, parabolic-like equations.

**Introduction.** In analyzing the evolution of rarefied plasmas, for those times less than the binary collision times, the Vlasov-Poisson system of equations is used for the relevant kinetic equations, in case the magnetic fields vary slowly. Such a model consists of a Liouville equation, i.e., a nonlinear hyperbolic conservation law, coupled with Poisson's equation for determining the self-consistent electrostatic or gravitational forces. For more realism, Poisson's equation can be replaced by Maxwell's equations for including electromagnetic effects. Collisionless plasma models have been the object of much mathematical study the last 10 years (cf., e.g., [1, 2, 7–14, 23–26] for a sample of relevant works).

The model of “collisionless plasmas”, especially in the applied contexts of controlled fusion, and of laser fusion, is a highly idealized one. A way to incorporate collisional effects of a plasma with the background material (e.g., a plasma system in a thermal bath or “reservoir”) is to consider the motion of an individual particle as Brownian motion caused by collisions with the background medium. This conceptual model is then analogous to the classical description