

Frequency Domain Design and Analytic Selections

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ABSTRACT. This paper analyzes a problem in which one optimizes an objective function over the space A of functions analytic on the disk. We demonstrate existence, regularity and uniqueness of solutions.

1. Introduction. This paper analyzes a problem in which one optimizes an objective function over the space A_N of vector-valued functions $f = (f_1, \dots, f_N)$ defined on the unit circle, $\partial\mathbf{D}$, where each coordinate function f_j extends to be analytic on the entire disk \mathbf{D} and continuous on its closure $\bar{\mathbf{D}}$. The objective we optimize is described in terms of a nonnegative continuous function Γ defined on $\partial\mathbf{D} \times \mathbf{C}^N$. Let

$$\gamma_A^* = \inf_{f \in A_N} \sup_{\vartheta} \Gamma(e^{i\vartheta}, f(e^{i\vartheta})).$$

The central concern of this paper is finding conditions on Γ primarily for $N = 1$ that guarantee the existence, uniqueness, and continuity of solutions $f^* \in A_N$ for which

$$\text{(OPT)} \quad \sup_{\vartheta} \Gamma(e^{i\vartheta}, f^*(e^{i\vartheta})) = \gamma_A^*.$$

This type of problem is central to frequency domain system design problems where stability is a key constraint. In particular, it is important to the area of H^∞ -control [H1, H2] and [Fr]. The basic physical idea is simple. Recall that a linear time invariant system has a frequency response function F defined on the imaginary axis and that the system is stable if F has no poles in the closed right half plane (R.H.P.). Behavior of the system when excited with a pure sine wave of frequency ω is determined by $F(i\omega)$. The following often occurs in a design procedure. We are required to build a system S but part of the system is given (we are stuck with it), part of the system is designable (denote its frequency response function by f). The performance of the system S at frequency ω is a function $\Gamma(\omega, f(i\omega))$ which depends on ω and on one's choice of the designable