

Addenda
“The Conormal Derivative Problem for
Non-uniformly Parabolic Equations”
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In this note we indicate some simple, useful extensions of the results in [3].

First we observe that the results in [3] are valid under weaker regularity hypotheses than given there: It is enough that u and Du be in L^∞ , D^2u and u_t be in L^2 , and $\partial\Omega$ be in $C^{1,1}$. The proofs are changed only in that phrases such as “all $h \in (0, T)$ ” are replaced by “almost all $h \in (0, T)$ ” and the corresponding suprema are replaced by essential suprema once we establish (1.5) in Lemma 1.1 of [3] under the weaker regularity hypothesis on $\partial\Omega$ with $\eta \in L^\infty$ and $D\eta \in L^2$ instead of $\eta \in L(Q_T)$. By means of a partition of unity, we may assume that η vanishes except in a small neighborhood of $\partial\Omega$ on which $|\gamma| = 1$. Since

$$\eta\gamma \in L_0^* = L^\infty(Q_T) \cap L^2((0, T); W_0^{1,2}(\Omega)),$$

we may approximate $\eta\gamma$ and therefore $\eta\gamma\gamma$ in L^* by functions in $L(Q_T)$ which vanish on $\partial\Omega$, using the obvious definition of L^* . Similarly, we can approximate $\eta - \eta\gamma\gamma$ in L^* by functions in $L(Q_T)$ which are orthogonal to γ . Adding together these approximating functions gives a sequence of $L(Q_T)$ vector-valued functions which are tangential on $\partial\Omega$ and approach η in L^* . Now [3, (1.5)] holds with η replaced by the approximating sequence and the integrals all converge to the corresponding ones with η .

Next we observe that the form of the estimate in [3, Theorem 4.1] can be improved if [3, (4.1b)] is strengthened to

$$(4.1b)' \quad v^2 \leq \beta_{12}^2 Du \cdot A.$$

If we note that (cf. [3, p. 47])

$$\begin{aligned} (wt^*\zeta)^{q-1} \frac{\zeta(v_1 - \tau_1)^2}{w} &\leq (wt^*\zeta)^{q-2} (4\rho)^2 \\ &\leq (4\beta_{12}\rho)^2 (wt^*\zeta)^{q-2} Du \cdot A, \end{aligned}$$