

Existence of Proper Mappings from Domains in \mathbf{C}^2

ELISABETTA BARLETTA & ERIC BEDFORD

ABSTRACT. Let $\Omega \subset \mathbf{C}^2$ be a pseudoconvex domain with real-analytic boundary. We give conditions for the existence of a complex manifold D and a branched, proper holomorphic mapping $f : \Omega \rightarrow D$. The essential point here is that f is required to have nonempty branch locus. In the strongly pseudoconvex case, a criterion is given in terms of the Moser normal form. In the weakly pseudoconvex case, we introduce the class of “weakly spherical” domains which, generally speaking, are the most likely to possess branched mappings. We then produce the corresponding normal form, which gives a necessary and sufficient condition for the existence of branched, proper mappings on weakly spherical domains.

1. Introduction. A typical way to construct a proper holomorphic mapping is to start with a holomorphic mapping $f : \Omega' \rightarrow \mathbf{C}^n$ and to find a domain $D \subset \mathbf{C}^n$ such that $f^{-1}(D)$ is relatively compact in Ω' . If Ω is a component of $f^{-1}(D)$, then the mapping $f : \Omega \rightarrow D$ is proper.

Here we wish to discuss the reverse situation: given a domain Ω , do there exist a domain D and a proper holomorphic mapping $f : \Omega \rightarrow D$? When D is a nonsingular variety, we define the branch locus as

$$V_f = \{z \in \Omega : f \text{ is not locally biholomorphic at } z\}$$

and we seek mappings for which V_f is nonempty. If there is a larger complex space (or manifold) \tilde{D} with $D \Subset \tilde{D}$, and if f extends smoothly to $\bar{\Omega}$, then f is a smooth mapping between the hypersurfaces $\partial\Omega$ and ∂D . The problem of studying the branching behavior of smooth mappings between hypersurfaces has been considered by several authors, and we refer the reader to [1, 3, 5, 17].

In this paper we study branching behavior by factoring by automorphisms. (This approach has been used earlier in [4, 10, 16, 24].) We say that $f : \Omega \rightarrow D$ is *factored by automorphisms* if there is a finite subgroup $\Gamma_f \subset \text{Aut}(\Omega)$ such that $f^{-1}f(z) = \{\chi(z) : \chi \in \Gamma_f\}$ for all $z \in \Omega$. Although a proper mapping cannot in