

On the Existence of Nonclassical Solutions for Two Classes of Fully Nonlinear Elliptic Equations

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ABSTRACT. We prove the existence of nonsmooth Lipschitz continuous solutions for two classes of fully nonlinear second order equations. The first is a class of equations involving the m^{th} elementary symmetric function of the Hessian and the second comprises the equations of prescribed m^{th} mean curvature, where in each case $m \geq 3$. This generalizes a result of Pogorelov for the Monge-Ampère equation.

1. Introduction. In [25], [26] Pogorelov constructed examples showing that convex generalized solutions of Monge-Ampère equations of the form

$$(1.1) \quad \det D^2u = g(x)$$

in a domain $\Omega \subset \mathbf{R}^n$, $n \geq 3$, need not be of class C^2 , even if g is positive and of class C^∞ , or even analytic. Our purpose here is to show that this absence of classical regularity is not confined to equations of Monge-Ampère type, but in fact occurs for a larger class of nonuniformly elliptic equations.

We shall be concerned with two classes of equations. The first of these comprises equations of the form

$$(1.2) \quad F_m(D^2u) = g(x, u, Du),$$

where F_m is defined by

$$(1.3) \quad F_m(D^2u) = S_m(\lambda_1, \dots, \lambda_n)^{1/m}.$$

Here $\lambda_1, \dots, \lambda_n$ are the eigenvalues of D^2u and for each integer m , $1 \leq m \leq n$, S_m is the m^{th} elementary symmetric function defined by

$$(1.4) \quad S_m(\lambda_1, \dots, \lambda_n) = \sum_{1 \leq i_1 < \dots < i_m \leq n} \lambda_{i_1} \cdots \lambda_{i_m}.$$