Cavitational Flow Past a Nonlinearly Elastic Panel

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1. Introduction. In this paper we study the cavitating, steady, irrotational, two-dimensional flow of an incompressible, inviscid fluid past a nonlinearly elastic cylindrical panel. See Figure 1.1. The large deformation of the panel is produced by the action of the net pressure field generated by the flow. The flow is assumed to have a prescribed velocity U and pressure P at infinity. The two-dimensionality of our problem means that the generators of the cylindrical panel in any deformed configuration remain perpendicular to the flow so that every section of the panel perpendicular to the generators suffers the same deformation. The equations for the deformation of a typical section are those for the planar deformation of a rod. We accordingly refer to such a section as a rod. We describe the deformation of these rods with a geometrically exact theory (cf. Antman and Rosenfeld (1978)) that accounts for flexure, compression, and shear. We allow the material properties of the rod to be described by a very general class of nonlinear constitutive relations. We demonstrate the global existence of a connected set of classical solution-parameter pairs of the governing equations.

Our approach is in the spirit of the work of Lanza and Antman (1990): We observe that U and the deformed shape of the rod uniquely determine the net pressure field on the rod. We substitute this pressure into the equilibrium equations for the elastic rod (formulated in Section 2), obtaining a system of functional-differential equations for the shape, which we transform into a fixed-point equation (in Section 3) by a procedure that is not entirely routine. To carry out the global analysis of this equation (in Section 3) we require detailed properties of the pressure field (obtained in Section 5), which depend on delicate properties of the flow. In obtaining these we emphasize only those aspects that differ significantly from the treatment of Lanza and Antman (1990). In formulating the flow problem (in Section 4), we exploit the rich body of results for cavitational flow past rigid obstacles. Our approach, however, differs in a number of significant respects from traditional formulations of the problem.