

Existence of Regular Solutions with Convex Level Sets for Semilinear Elliptic Equations with Non-monotone L^1 Nonlinearities Part II. Passage to the Limit

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ABSTRACT. We prove existence of solutions with convex level sets for certain semilinear elliptic differential equations in convex annuli. These solutions are shown to be more regular than is indicated by the general theory, and are limits of solutions to associated multiphase free boundary problems.

Introduction. In this work we study the boundary value problem

$$(1) \quad \begin{aligned} \Delta u &= f(u) \text{ on } R \\ u &= 0 \text{ on } \Gamma_{\text{ext}}, \quad u = M \text{ on } \Gamma_{\text{int}} \end{aligned}$$

where $R \subseteq \mathbb{R}^2$ is an annular domain with convex inner and outer boundaries Γ_{int} and Γ_{ext} respectively, and M is a strictly positive constant. The only restrictions we put on the nonlinearity f are that

$$(2) \quad f \in L^1(-\infty, \infty), \quad f \geq 0, \quad \text{and } f = 0 \text{ on } (-\infty, 0).$$

All results hold in \mathbb{R}^n as well if one assumes the free boundary conditions introduced in Section 3.

Our object is to demonstrate the existence of solutions which have convex level lines under considerably weaker assumptions on the nonlinearity f than have been treated in the literature. These solutions are significantly more regular than predicted by the general theory, see [B-B-C] for example. In the special case when f is monotone increasing solutions are unique so our result becomes a true regularity result. (Note in [C-S] it is proved that if f is continuous and monotone