

Curvatures, Functionals, Currents

G. ANZELLOTTI, R. SERAPIONI, I. TAMANINI

ABSTRACT. The starting point is the problem of minimizing functionals $\mathcal{F}(M)$, defined on surfaces M and depending on the curvatures of M . The point of view is that of the direct method in the Calculus of Variations. The key idea is to consider the graph G of the Gauss map of a surface M , and to consider functionals \mathcal{F} such that $\text{Area}(G) \leq c\mathcal{F}(M)$. Then one can use the known compactness results for integral currents to obtain convergent minimizing sequences of graphs G_j . The currents which are possible limits of such minimizing sequences and their generalized curvatures are investigated. Functionals defined on these classes of currents are studied.

0. Introduction.

(A) DESCRIPTION OF THE OBJECTIVES AND OF THE MAIN RESULTS. The starting point of this paper is the problem of minimizing functionals defined on submanifolds and involving curvatures. A few examples of classical functionals depending on curvatures are collected in Section (B) of this introduction. In this paper we are concerned with existence problems and we take the point of view of the Direct Method of the Calculus of Variations. More precisely, we shall consider the following questions:

- (i) what weak compactness properties have the minimizing sequences, provided reasonable coerciveness assumptions are made on the functional?
- (ii) what are the properties of the “surfaces” which are weak limits of smooth surfaces with uniformly bounded L^1 -norms of the curvatures? In particular, do such surfaces have first and second order differentiability properties, and can one still define “curvatures” for them?
- (iii) how curvatures, boundary conditions and topological type behave with respect to weak convergence?
- (iv) what is a good extension of the classical functionals to the larger spaces of “surfaces” where the existence can be obtained?