

Semi-Discretized Nonlinear Evolution Equations as Discrete Dynamical Systems and Error Analysis

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ABSTRACT. This paper is a continuation of our study on doubly nonlinear parabolic type equations that was initiated by a previous paper; it is however self-contained. We propose here to investigate a time discretization of the continuous problem by the Euler forward scheme where in addition to the standard existence, uniqueness and stability questions, we also address the problem of error estimate and the long time behavior of the solutions to the discrete problem. The existence of a compact attractor is proven and its Hausdorff dimension is estimated using CFT theory.

1. Introduction. In this paper we continue our study of the doubly nonlinear parabolic type equations in a different direction than the one initiated in Eden et al, (1989). The questions we would like to address are related with the consistency and the validity of the numerical solutions as approximations to these equations for long times. As a first step in this endeavour, we propose to study only the time discretization of the continuous problem by Euler forward scheme where, in addition to the standard existence, uniqueness and stability questions, we will consider some error estimates and the long time behavior of the solutions to the discrete problem. Our work is inspired on one hand by a wealth of research that is conducted on degenerate elliptic and parabolic equations such as Alt and Luckhaus (1983), Blanchard and Francfort (1988), Grange and Mignot (1972), Rakotoson (1987), Rakotoson and Temam (1987), and Raviart (1970), and their numerical counterparts, Magenes (1988), Nochetto and Verdi (1988), Verdi (1987) and Verdi and Visintin (1988); and on the other hand by the study of the infinite dimensional dynamical systems arising from dissipative partial differential equations, as a means of understanding the long time behavior of their solutions, see Constantin et al (1985) and Temam (1988). Even though we will not treat the fully discretized problem, we believe that the study of the time