

Lipschitz Approximation to Hypersurfaces, Harmonic Measure, and Singular Integrals

G. DAVID & D. JERISON

ABSTRACT. We study hypersurfaces satisfying a scale-invariant condition generalizing the chord-arc condition on planar curves. We show that, at all scales, a large fraction of the surface coincides with a Lipschitz graph. One consequence is that singular integral operators generalizing the Cauchy integral on curves are bounded on L^p ($1 < p < \infty$). Our construction shows that if the surface bounds a nontangentially accessible domain, then the Lipschitz graph can be taken to be inside the domain. We deduce that harmonic measure and surface measure are mutually absolutely continuous (in the scale-invariant sense of the Muckenhoupt class A_∞).

Introduction. In [S], S. Semmes introduced a class of hypersurfaces of \mathbf{R}^{d+1} , for which the Clifford-valued analogue of the Cauchy kernel defines a bounded operator on L^2 for the surface measure. We shall define a slightly more general class of hypersurfaces \mathcal{S} for which the same result holds. Our proof will consist of finding, at all scales, “big pieces of Lipschitz graphs” in \mathcal{S} , and then using “good λ inequalities.” This proof is shorter than the argument in [D2], and also gives more: if \mathcal{U} is one of the connected components of the complement of \mathcal{S} , and is a nontangentially accessible domain, then one can ask the Lipschitz graph to be on the same side of \mathcal{S} as \mathcal{U} . Using B. Dahlberg’s result on harmonic measure on Lipschitz domains [DH] and a localization argument [JK], one then proves that the harmonic measure on $\partial\mathcal{U}$ is absolutely continuous (and even in the Muckenhoupt class A_∞) with respect to d -dimensional surface measure.

When $d = 1$, this extends a result of Lavrentiev [L] for chord-arc domains. Also, we cannot hope for a better A_p -class than A_∞ , even when $d = 1$ and \mathcal{S} is a chord-arc curve (see [Je] or [JZ] for counterexamples).

The main geometric construction is given in Section 1, and the results are stated in Section 2.