

Composition Operators on S_a Spaces

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1. Introduction. For a real number a , we say that the complex formal power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ belongs to the set S_a if $\sum |a_n|^2 (n+1)^{2a} < \infty$. By [14], S_a is a Hilbert space with the inner product

$$(f, g)_a = \sum_{n=0}^{\infty} a_n \bar{b}_n (n+1)^{2a}$$

where $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$. Also, by Theorem 10 from [14], the spaces S_a are spaces of functions analytic in the unit disc D . Note that S_0 is the classical Hardy space H^2 , $S_{-1/2}$ is the Bergman space, and $S_{1/2}$ is the Dirichlet space.

If H is a Hilbert space of functions defined on the unit disc D and the function φ maps D into itself, we define the composition operator C_φ on H by

$$C_\varphi f = f \circ \varphi.$$

The study of composition operators began in 1968 with the work of E. Nordgren (see [7]). From then on, most work was on the properties of composition operators on the space H^2 (see for example [11], [13], [2], [8], [3], and [9]) although there were some results obtained in a more general setting. (See [1], [16], [6], [12], and [5].)

In this article we try to give characterizations of bounded and compact composition operators on spaces S_a with $a > 1$, varying the results of B. MacCluer and J. Shapiro obtained in [6] and [5] on spaces S_a when $a \leq 1$.

In the case when $1 < a \leq \frac{3}{2}$ we give some one-way results about the boundedness of composition operators on the spaces S_a . For example, if φ is in S_a and $\|\varphi\|_\infty < 1$, or φ' is in the multiplier space $M(S_{a-1})$ of S_{a-1} , then C_φ is bounded on S_a . The other results are in terms of Carleson measures and we shall state them in Section 2.

When $a > \frac{3}{2}$, Theorem 1 states that all composition operators induced by a function which is in the space S_a are bounded.