

Spectra and Resolvents of Certain Polynomials in the Irrational Rotation Algebra

NORBERT RIEDEL

1. Let α be an irrational number and let \mathcal{A} be the irrational rotation C^* -algebra associated with α , i.e., there are unitaries u and v in \mathcal{A} satisfying the relation $uv = e^{2\pi\alpha i}vu$ such that \mathcal{A} is generated by u and v . The purpose of this note is to investigate spectral properties of elements of the form

$$au + bu^* + cv + dv^*,$$

where a, b, c, d are non vanishing real numbers. Alternatively, we consider real multiples of elements of the form

$$(1.1) \quad \gamma u + \gamma^{-1}u^* + \beta(\delta v + \delta v^*),$$

$(\beta, \gamma, \delta \in \mathbf{R} \setminus \{0\})$. For $\gamma = \delta = 1$ we have almost Mathieu operators. In [1] we have derived a system of linear equations that, among other things, provides for the characterization of elements in the spectrum of almost Mathieu operators in terms of uniformly bounded double sequences of Fourier coefficients of certain functionals. (Here, as well as in the sequel the terminology of Fourier analysis refers to expansions in the monomials $e^{-pq\alpha i}u^p v^q$.) What we intend to show is, that this system of equations (essentially) encodes the complete information needed to determine the spectra and resolvents for operators of the form (1.1). We shall distinguish between two types of components in the resolvent set, one type that arises with the resolvent of an almost Mathieu operator (we shall refer to these components as being of Type I), and another type, for which the resolvent does not relate to the resolvent of an almost Mathieu operator (we shall call those components to be of Type II). The unbounded component is always of Type I. We have explicit recursions for the Fourier coefficients of the resolvent on components of Type II. Furthermore, we shall see that the closed subalgebra of \mathcal{A} generated by the values the resolvent takes on components of Type II (for a fixed set of parameters β, γ, δ) is isomorphic to a subalgebra of the algebra of formal power series in one variable which converge in an open disk. If α satisfies a certain diophantine condition and β is sufficiently large then the components of Type II of the resolvent set of $\gamma u + \gamma^{-1}u^* + \beta(\delta v + \delta^{-1}v^*)$ always cover a dense set of the spectrum of $u + u^* + \beta(v + v^*)$.