

Homeomorphic Disks in the Spectrum of H^∞

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ABSTRACT. In the first part of this paper we give several descriptions of those analytic disks $P(m)$ in the spectrum of H^∞ which are homeomorphic to the unit disk. The main result will be a complete characterization of the zero sets of interpolating Blaschke products as well as the interpolating sequences on these homeomorphic disks. In the second part we show that the Hoffman map L_m associated with a homeomorphic disk has a homeomorphic extension to the set G of all non-trivial points in H^∞ . Finally, in the third section, we answer several questions of Guillory on the characterization of thin Blaschke products.

0. Introduction. Let H^∞ be the Banach algebra of all bounded analytic functions in the open unit disc $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$ and let $M(H^\infty)$ denote its maximal ideal space. For $m, x \in M(H^\infty)$, let $\rho(m, x) = \sup\{|f(x)| : f \in H^\infty, f(m) = 0, \|f\| = 1\}$ denote the pseudohyperbolic distance of the points m and x in $M(H^\infty)$. By Schwarz-Pick's lemma $\rho(z, w) = \left| \frac{z-w}{1-\bar{z}w} \right|$ if $z, w \in \mathbf{D}$. Let

$$P(m) = \{x \in M(H^\infty) : \rho(m, x) < 1\}$$

be the Gleason part of $m \in M(H^\infty)$. Defining m to be equivalent to x , $m \sim x$, if $\rho(m, x) < 1$, one can show ([5], p. 402) that \sim is an equivalence relation in $M(H^\infty)$. Thus the Gleason parts of two points are either disjoint or equal.

A Gleason part P is called an *analytic disk* if there exists a continuous, bijective map L of \mathbf{D} onto P such that $\hat{f} \circ L$ is analytic in \mathbf{D} for every $f \in H^\infty$, where \hat{f} denotes the Gelfand transform of $f \in H^\infty$. In his famous paper [11] K. Hoffman showed that any Gleason part $P(m)$ in $M(H^\infty)$ is either a single point or an analytic disk. Moreover, the latter occurs if and only if $m \in \mathbf{D}$ or