

Traveling Waves of a Perturbed Diffusion Equation Arising in a Phase Field Model

R. A. GARDNER & C. K. R. T. JONES

ABSTRACT. The existence of traveling waves and their stability are well understood for scalar nonlinear diffusion equations. Motivated by an equation occurring in a phase field model, we study the effect of adding terms involving higher order derivatives to such an equation. An algebraic condition on the coefficients is given that guarantees both the existence of travelling waves for the perturbed equation and their stability. The methods of geometric singular perturbation theory are used for the existence. These are combined with topological methods to obtain the stability.

1. Introduction. In [4] Caginalp and Fife propose a model for studying phase fields that includes terms with even order spatial derivatives of arbitrarily high order. Although their model is a system, a natural starting point is to consider the isothermal case in which heat does not play a role. This is achieved by setting the latent heat to zero and assuming that the initial temperature is constant. One then obtains a scalar equation which generalises the well-known bistable equation by including higher order derivatives. The solutions of interest for this equation are traveling waves which represent transitions from one phase to another. This paper is devoted to a study of the existence and stability of such traveling waves.

The aim is to understand how these higher derivative terms affect the structure of the second order equation. If one progressively adds on terms, starting with a fourth order term, it is easily seen that only the addition of every second term gives a well-posed PDE. Indeed, in the general $2n^{\text{th}}$ order problem

$$(1) \quad \frac{\partial \varphi}{\partial t} = a_{2n} \frac{\partial^{2n} \varphi}{\partial x^{2n}} + a_{2n-2} \frac{\partial^{2n-2} \varphi}{\partial x^{2n-2}} + \cdots + a_2 \frac{\partial^2 \varphi}{\partial x^2} + f(\varphi)$$