

Homogeneous Complex Monge–Ampère Equations and Algebraic Embeddings of Parabolic Manifolds

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ABSTRACT. Let M^n be a Stein manifold, and let $\tau : M \rightarrow [0, R^2)$, $0 < R \leq \infty$, be a strictly plurisubharmonic exhaustion. The pair (M, τ) is said to be *strictly parabolic at infinity* if $u = \log \tau$ satisfies the Monge–Ampère equation $(\partial\bar{\partial}u)^n \equiv 0$ outside some compact set K . Additionally, (M, τ) is of *Reinhardt type* if τ can be written locally as a Reinhardt function on $M - K$. The set $M - K$ admits a Monge–Ampère foliation in a well-known way. With an assumption on the holomorphic twist of this foliation when $R = \infty$ we prove a curvature estimate, which, via a theorem due to Demailly, implies that M embeds algebraically into \mathbb{C}^{2n+1} . When $R \leq \infty$ and $K = \tau^{-1}(0)$ we prove, under a weaker regularity assumption than usual, that M is biholomorphic to the ball in \mathbb{C}^n of radius R .

1. Introduction.

Outline. Let M be a connected, complex manifold of dimension n , and let $\tau : M \rightarrow [0, R^2)$, $0 < R \leq \infty$, be a strictly plurisubharmonic (spsh) exhaustion. Stoll [S2] defined a *strictly parabolic manifold* to be one in which the function $u = \log \tau$ satisfies the homogeneous Monge–Ampère equation $(\partial\bar{\partial}u)^n = 0$ on $M - \tau^{-1}(0)$. He proved that if τ is C^∞ , the only example is the R -ball in \mathbb{C}^n with the standard exhaustion $\tau_0(z) = |z|^2$. Burns [B1] gave an alternate proof of this and, using a curvature estimate, drew the same conclusion when τ is C^5 and $R = \infty$. It is unknown if the conclusion holds when τ is C^5 and $R < \infty$.

In this paper we follow a generalization of the notion of strictly parabolic manifold due to Burns [B2], [F2]. Let M , τ , and u be as above with $R = \infty$, except that $(\partial\bar{\partial}u)^n = 0$ is required to hold only outside some compact subset of M . We will say that M , together with τ , is *strictly parabolic at infinity*.