

# *The Initial-Dirichlet Problem for a Fourth-Order Parabolic Equation in Lipschitz Cylinders*

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ABSTRACT. We study the initial-Dirichlet problem for the fourth-order parabolic equation  $\partial_t + \Delta^2$  on a cylinder  $(0, T) \times D$  where  $D \subset \mathbf{R}^n$  is a bounded domain with Lipschitz boundary. On the lateral boundary  $(0, T) \times \partial D$  we specify the boundary value of the function to lie in a Sobolev space of functions having one spatial derivative and one quarter of a time derivative in  $L^2$ . The normal derivative is taken to lie in  $L^2$ . We only consider the case of homogeneous initial data. With these assumptions, we show that there exists a unique solution whose spatial gradient has parabolic maximal function in  $L^2$ . The main interest of these results lies in the weak smoothness assumptions on the lateral boundary.

**Introduction.** Let  $D \subset \mathbf{R}^n$  be a bounded domain with Lipschitz boundary. We consider the initial-Dirichlet problem for the equation  $\partial_t + \Delta^2$  in  $(0, T) \times D$ :

$$(iDP) \quad \begin{cases} \partial_t u + \Delta^2 u = 0, & \text{in } (0, T) \times D \\ u(0, X) = 0, & X \in D \\ u(s, Q) = f(s, Q), & \text{on } (0, T) \times \partial D \\ \partial_\nu u(s, Q) = g(s, Q), & \text{on } (0, T) \times \partial D, \end{cases}$$

where  $\partial_\nu$  denotes differentiation in the direction of the outward unit normal  $\nu$ . The boundary value  $f$  will be taken from the Sobolev space

$$L^{2,1/4}((0, T); L^2(\partial D)) \cap L^2((0, T); L^{2,1}(\partial D))$$

and  $g$  will be taken from  $L^2((0, T) \times \partial D)$ . Under these assumptions, we will produce a unique solution of (iDP) and establish an  $L^2$ -estimate near the boundary for the spatial gradient and a quarter time derivative. Our estimates will be