

# *New A Priori Estimates for the Stokes Equations in Exterior Domains*

HIDEO KOZONO & HERMANN SOHR

ABSTRACT. Consider the stationary Stokes equations

$$-\Delta u + \nabla p = f, \quad \operatorname{div} u = g \text{ in } \Omega, \quad u = \varphi \text{ on } \partial\Omega,$$

where  $\Omega \subset \mathbf{R}^n$  ( $n \geq 3$ ) is an exterior domain. In the present paper, we develop new a priori estimates in  $L^q$ -spaces for both weak and strong solutions including higher regularities. Asymptotic behavior for these solutions is also investigated. Our estimates may be regarded as generalizations of Cattabriga's and Solonnikovs' in 3-dimensional bounded domains to  $n$ -dimensional ( $n \geq 3$ ) exterior domains.

## 1. Introduction

Let  $\Omega$  be an exterior domain in  $\mathbf{R}^n$  ( $n \geq 3$ ), i.e.,  $\Omega$  is a domain whose complement is nonempty and compact. We assume that the boundary  $\partial\Omega$  of  $\Omega$  is of class  $C^{2+\mu}$  with some  $\mu \in (0,1)$ . Consider the boundary value problem for the Stokes equations in  $\Omega$ :

$$(S) \quad -\Delta u + \nabla p = f, \quad \operatorname{div} u = g \text{ in } \Omega, \quad u = \varphi \text{ on } \partial\Omega,$$

where  $u = u(x) = (u^1(x), u^2(x), \dots, u^n(x))$  and  $p = p(x)$  denote the unknown velocity vector and pressure of the fluid, respectively, while  $f = f(x) = (f^1(x), f^2(x), \dots, f^n(x))$ ,  $g = g(x)$  ( $x \in \Omega$ ) and  $\varphi = \varphi(\xi) = (\varphi^1(\xi), \varphi^2(\xi), \dots, \varphi^n(\xi))$  ( $\xi \in \partial\Omega$ ) denote the given functions in  $\Omega$  and  $\partial\Omega$ , respectively.  $\Delta$  is the Laplacian and  $\nabla$  is the gradient.

The purpose of the present paper is to give an essentially complete existence and regularity result on (S) in exterior domains. In particular, we shall show the existence of a weak solution of (S) for  $f$  in  $H^{-1,q}$  and prove its regularity