

# *Oscillatory Integrals and Regularity of Dispersive Equations*

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**1. Introduction.** This paper is concerned with oscillatory integrals and their relationship with the local and global smoothing properties of dispersive equations. Also we shall study some applications of these smoothing properties to nonlinear problems and their link with restriction theorems for the Fourier transform and pointwise convergence results.

We begin by considering the initial value problem (IVP) for the free Schrödinger equation

$$(1.1) \quad \begin{cases} \partial_t u - i\Delta u = 0 & x \in \mathbb{R}^n, t \in \mathbb{R} \\ u(x, 0) = u_0(x). \end{cases}$$

whose solution  $u(\cdot)$  is given by

$$(1.2) \quad u(x, t) = W(t)u_0(x) = \int e^{i(-t|\xi|^2 + x\xi)} \hat{u}_0(\xi) d\xi$$

where  $\hat{u}_0$  denotes the Fourier transform of  $u_0$ . Since  $\{W(t)\}_{-\infty}^{\infty}$  is a unitary group in any Sobolev space  $H^s(\mathbb{R}^n)$ , no regularizing effects of the solutions can be expected in  $H^s(\mathbb{R}^n)$ . Nevertheless, the result given by R. S. Strichartz [Sc] (motivated by an earlier result of I. E. Segal [Sg] for the Klein-Gordon and wave equations) shows that if  $u_0 \in L^2(\mathbb{R}^n)$ , the solution  $u(x, t)$  in (1.2) belongs to  $L^{2+4/n}(\mathbb{R}^{n+1})$  for a.e.  $t$ . More precisely,

**Theorem 1.1** [Sc]. *Let  $u(x, t)$  be the solution of the IVP (1.1). Then*

$$\left( \iint |u(x, t)|^{2(n+2)/n} dx dt \right)^{n/2(n+2)} \leq c \left( \int |u_0|^2 dx \right)^{1/2}.$$

The proof of this theorem is based on a previous work of P. Tomas [T] and E. M. Stein [St, p. 326] about the restriction of the Fourier transform to the