

On the Life Span of the Analytic Solutions to Quasilinear Weakly Hyperbolic Equations

P. D'ANCONA & S. SPAGNOLO

ABSTRACT. Consider the second order nonlinear Cauchy problem

$$u_{tt} = F(t, x, u, u_x, u_{xx}), \quad u(0, x) = \varepsilon \varphi(x), \quad u_t(0, x) = \varepsilon \psi(x)$$

assuming that it is *weakly hyperbolic* at $u = u_x = u_{xx} = 0$ (i.e., the matrix $F_{u_{ij}}(t, x, 0, 0, 0)$ is nonnegative definite) and that $F(t, x, 0, 0, 0) = 0$; assume moreover that F, φ, ψ are real analytic in all their variables (possibly excepted t) and periodic in x : it is then known, by the Cauchy-Kowalewski theorem, that a local solution exists on some strip $[0, T_\varepsilon[\times \mathbf{R}^n$.

Here we prove that $T_\varepsilon \rightarrow +\infty$ for $\varepsilon \rightarrow 0$. Moreover, in the autonomous case (F independent of t) we can give the estimate $T_\varepsilon \geq \mu \log \log(\frac{1}{\varepsilon})$.

1. Introduction. Let us consider the initial value problem

$$(1) \quad u_{tt} - \sum_{i,j}^{1,n} a_{ij}(t, x, u, u_x) u_{x_i x_j} = f(t, x, u, u_x) \quad (t \geq 0)$$

$$(2) \quad u(0, x) = \varepsilon \varphi(x), \quad u_t(0, x) = \varepsilon \psi(x)$$

where

$$(3) \quad f(\cdot, \cdot, 0, 0) = 0.$$

Assume that the functions a_{ij}, f, φ, ψ are continuous in t and *analytic* in all the other arguments; assume moreover that these functions are 2π -periodic in x_1, \dots, x_n (see, however, Remark 4 for the non periodic case).