

Quasiconformal Mappings and Capacity

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ABSTRACT. This paper deals with the analytic theory of quasiconformal mappings in even dimensions. We give estimates for the exponent of integrability of a K -quasiconformal mapping, arising from norm estimates of the higher dimensional Beurling–Ahlfors transform, and relate these to the distortion of Hausdorff and capacitory dimension. A new estimate is found for the Marcinkiewicz constant involved in the planar measure distortion inequality. This is used to study removable singularities of planar quasiregular mappings by considering those sets whose image under such mappings has zero analytic capacity.

This paper can be viewed as a sequel to our paper [IM1] where we have found new aspects of the analytic theory of quasiregular mappings in even dimensions following some ideas from the quasiconformal Yang–Mills theory developed by Donaldson and Sullivan [DS]. In recent years there have been intensive studies of the analytic and geometric properties of quasiconformal mappings. One of the most fundamental results used in the analytic theory of quasiconformal mappings is Gehring’s Theorem [G1]. It asserts that the differential of a quasiconformal mapping of \mathbb{R}^n (apriori assumed locally L^n -integrable) has improved integrability properties depending upon the dilatation. There is strong motivation to determine precisely the best exponent of integrability of the differential in terms of the dimension and dilatation. Before outlining our results, we recall a couple of definitions.

Definition. Let $\Omega \subset \mathbb{R}^n$ be a domain and $f : \Omega \rightarrow \mathbb{R}^n$ a mapping of Sobolev class $W_{n,\text{loc}}^1(\Omega)$. We let $Df(x)$ denote the *Jacobian matrix* and $J(x, f)$ the *Jacobian determinant*. Then f is said to be K -*quasiregular*, $1 \leq K < \infty$, if

- (i) $J(x, f) \geq 0$ a.e. or $J(x, f) \leq 0$ a.e.
- (ii) $\max_{|h|=1} |Df(x)h| \leq K \min_{|h|=1} |Df(x)h|$ a.e.