

*Local Existence of Multivalued Solutions to
Analytic Symplectic Monge-Ampère Equations
(The nondegenerate and type changing cases)*

MAREK KOSSOWSKI

In memory of Danuta Kossowska

ABSTRACT. The objective of this paper is to use multivalued solutions to establish the local existence or nonexistence of specific types of weak single-valued solutions. Our main result is that for the class of symplectic Monge-Ampère equations, this existence is independent of linearized symbol type and is controlled by the cotangent characteristic variety.

Introduction. This paper concerns the local existence and qualitative properties of multivalued solutions for a class of determined nonlinear analytic PDE. One objective is to use multivalued solutions to establish the local existence or nonexistence of specific types of weak single-valued solutions (i.e., locally defined single-valued functions which are analytic on the complement of a closed analytic subset where they are C^0 but fail to be C^1). In geometric applications multivalued solutions are of their own interest since their graphs correspond to smooth hypersurfaces with singular (i.e., non-immersed) points. Our main observation is that the existence of such solutions is independent of the linearized symbol type (i.e., the elliptic, parabolic, or hyperbolic type of PDE) and is controlled by the cotangent characteristic variety.

Our approach is to view a symplectic Monge-Ampère PDE as an auxiliary analytic differential 2-form on the cotangent bundle of the plane, $\pi : T^*\mathbf{R}^2 \rightarrow \mathbf{R}^2$. A solution is defined to be an immersed open disk $s : D \rightarrow T^*\mathbf{R}^2$ which annihilates both the canonical and auxiliary 2-forms. A solution is called multivalued if π^*s fails to have maximal rank. Such a solution may be thought of as a collection of locally defined single-valued solutions which are “held together” by $s(D) \subset T^*\mathbf{R}^2$. Any such multivalued s has a graph in $\mathbf{R}^2 \times \mathbf{R}$ which is, in general, a non-immersed hypersurface. For special s it is possible to excise from this graph a weak single-valued function. Thus the existence of such a special multivalued solution indirectly implies the existence of a locally defined weak