

The Space of Minimal Submanifolds for Varying Riemannian Metrics

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ABSTRACT. Let N be a smooth manifold and let \mathcal{M} be the space of pairs (γ, M) where γ is a smooth riemannian metric on N and $M \subset N$ is a minimal (with respect to γ) submanifold. This paper shows that \mathcal{M} is a Banach manifold and that the projection $\Pi : (\gamma, M) \mapsto \gamma$ (from \mathcal{M} to the space of metrics on N) is a smooth map of Fredholm index 0. Applications include a proof that every metric of positive ricci curvature on S^3 admits a minimal embedded torus.

Contents

0. Introduction	161
1. General Results about the Dependence of Critical Points on Parameters	164
2. Minimal Submanifolds	177
3. Perturbing Continuous Families of Minimal Submanifolds ..	181
4. Examples	185
5. The Mapping Degree of Π	190
6. Minimal Surfaces in 3-manifolds of Positive Ricci Curvature	191
7. Immersed Surfaces, Constant Mean Curvature Surfaces, and Parametric Elliptic Functionals	196

Introduction. Let M and N be smooth riemannian manifolds such that M is compact and such that the dimension of M is less than the dimension of N , and let Γ be an open subset of C^q riemannian metrics on N ($q \geq 3$). If u is a map from M to N , let $[u]$ denote the set of all maps $u \circ \varphi$ where $\varphi : M \rightarrow M$ is a diffeomorphism. (Thus the elements of $[u]$ are all parametrizations of the same surface $u(M)$.) This paper is a study of the space \mathcal{M} of ordered pairs $(\gamma, [u])$