

# *Evolution of Nonparametric Surfaces with Speed Depending on Curvature, I. The Gauss Curvature Case.*

VLADIMIR OLIKER

**1. Introduction.** Let  $\Omega$  be a bounded strictly convex domain in  $R^n$ ,  $n \geq 2$ , with smooth boundary  $\partial\Omega$ , and  $u(x, t)$ ,  $x \in \bar{\Omega}$ ,  $t \in [0, \infty)$ , a solution of the initial boundary value problem

$$u_t = \frac{\det[u_{ij}]}{(1 + |\nabla u|^2)^\beta} \text{ in } \Omega \times (0, \infty), \beta = \text{const} \geq 0,$$

(1.1)  $u(x, t)$  is strictly convex (downward) for each  $t \geq 0$ ,

$$u(x, t) = 0 \text{ in } \partial\Omega \times [0, \infty),$$

where

$$u_t := \frac{\partial u}{\partial t}, u_{ij} := \frac{\partial^2 u}{\partial x^i \partial x^j}, \nabla u := \text{grad}_x u, |\nabla u|^2 = \langle \nabla u, \nabla u \rangle.$$

Geometrically, the graphs  $(x, u(x, t))$ ,  $(x, t) \in \bar{\Omega} \times [0, \infty)$  may be viewed as a family of (hyper-) surfaces evolving in space  $\mathbf{R}^{n+1}$  whose relative boundaries  $(x, u(x, t))|_{\partial\Omega}$  remain fixed. When  $\beta = \frac{(n+1)}{2}$  the normal speed of the point  $(x, u(x, t))$  (that is, the projection of the velocity vector  $(0, u_t(x, t))$  on the direction of the unit normal at this point) is equal to the Gauss curvature of the graph.

The main objective of this paper is to analyze the asymptotic behaviour of smooth convex solutions of (1.1). Appropriate existence results for (1.1) can be extracted (with some work) from the results in the book by Krylov [Kr]. Here we give sharp estimates of the rate at which  $u(x, t) \rightarrow 0$  as  $t \rightarrow \infty$ . Our estimates allow to deduce various geometric properties of the flow described by  $u(x, t)$ . For example, we show that the flow ‘picks up’ the symmetries of the domain