

On a Problem of A. Beardon and K. Stephenson

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ABSTRACT. In the paper [1] by A. Beardon and K. Stephenson, “The uniformization theorem for circle packings”, there is a list of open questions. The first one asks whether two infinite circle packings of the unit disk which have bounded degree (that is, the number of circles tangent to any circle is uniformly bounded) and which have the same combinatorics are equivalent under a Möbius transformation of the disk. In the present paper it is shown that the methods of D. Sullivan [5] and Z.-X. He [2] can be used to prove that the answer is affirmative.

There is a list of open questions at the end of [1]. The first open question asks whether an infinite circle packing of the unit disk having bounded degree is essentially uniquely determined by its combinatorics. The purpose of this note is to show that the methods of D. Sullivan [5] and Z.-X. He [2] can be applied to give an affirmative answer. (Added in proof—February 5th, 1991: A recent result of Oded Schramm [*Rigidity of infinite circle packings*, J. Amer. Math. Soc. (to appear)] shows that the answer remains affirmative even when the hypothesis of bounded degree is removed.)

Let T be an infinite triangulation of the open unit 2-disk \mathbb{D} . A collection P of closed disks in \mathbb{D} is said to be a *circle packing of \mathbb{D} with combinatorics T* if the disks of P have disjoint interiors and if the set of line segments which join the centers of every pair of tangent disks of P determine a triangulation of \mathbb{D} which is isotopic to T . Note that the disks of P fill \mathbb{D} completely in the sense that every point of \mathbb{D} lies in one of the disks of P or else lies in one of the (circular triangle) interstices formed by triples of mutually tangent disks of P . We shall prove the following result.

Theorem. *If T has bounded degree (i.e., there is an integer n such that at most n edges meet at each vertex) and if P and P' are circle packings of \mathbb{D} with combinatorics T , then there is a Möbius transformation of \mathbb{D} which sends P to P' .*