

# *Foliations on a Riemannian Manifold and Ehresmann Connections*

NAOYUKI KOIKE

**ABSTRACT.** The purpose of this paper is to obtain two new sufficient conditions for the normal bundle of the foliation on a Riemannian manifold to be an Ehresmann connection for the foliation. In addition, we give examples illustrating the need of the various hypotheses in the main result.

**Introduction.** C. Ehresmann [6] defined a connection in a fibre bundle to be a complementary distribution to the fibres on the total space satisfying the condition that every curve in the base space can be lifted horizontally to the total space. R. A. Blumenthal and J. J. Hebda [3] generalized the notion for a foliated manifold. They called it an Ehresmann connection for the foliation. Also, they [3, 5] obtained some sufficient conditions for the normal bundle of a foliation on a Riemannian manifold to be an Ehresmann connection for the foliation.

In this paper, we will obtain some new sufficient conditions for the normal bundle to be an Ehresmann connection, which are generalizations of some of the ones given in [3, 5].

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**1. Preliminaries.** Throughout this paper, unless otherwise mentioned, we assume that all objects are smooth and all manifolds are connected. Let  $M$  be an  $n$ -dimensional manifold and  $E$  a subbundle of the tangent bundle  $TM$  (i.e., a distribution on  $M$ ). The fibre of  $E$  over  $x \in M$  is denoted by  $E_x$  and the dual bundle of  $E$  by  $E^*$ . The space of cross-sections of  $E$  is denoted by  $\Gamma(E)$ . For a piecewise smooth curve  $\alpha : [0, 1] \rightarrow M$ , we define a piecewise smooth curve  $\alpha^{-1} : [0, 1] \rightarrow M$  by  $\alpha^{-1}(t) := \alpha(1 - t)$  for  $t \in [0, 1]$  and a piecewise smooth curve  $\alpha|_{[t_0, t_1]} : [0, 1] \rightarrow M$  by  $\alpha|_{[t_0, t_1]}(t) := \alpha(t_0 + (t_1 - t_0)t)$  for  $t \in [0, 1]$ , where  $[t_0, t_1] \subset [0, 1]$ . Also, for two piecewise smooth curves  $\alpha : [0, 1] \rightarrow M$