

Structure Theorems for Constant Mean Curvature Surfaces Bounded by a Planar Curve

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1. Introduction. A circle C in \mathbb{R}^3 is the boundary of two spherical caps of constant mean curvature H for any positive number H , which is at most the radius of C . It is natural to ask whether spherical caps are the only possible examples. Some examples of constant mean curvature immersed tori by Wente [6] indicate that there are compact genus-one immersed constant mean curvature surfaces with boundary C that are approximated by compact domains in Wente tori; however, this has not been proved. Still one has the conjecture:

Conjecture 1. *A compact constant mean curvature surface bounded by a circle is a spherical cap if either of the following conditions hold:*

- (1) *The surface has genus 0 and is immersed;*
- (2) *The surface is embedded.*

If M is a compact embedded constant mean curvature surface in \mathbb{R}^3 with boundary C and M is contained in one of the two halfspaces determined by the plane containing C , the Alexander reflection method [1] immediately proves M has the planar reflectional symmetries of C ; hence, M is a surface of revolution. Since the only compact constant mean curvature surfaces of revolution are spherical caps (by Delaunay's classification [2] of constant mean curvature surfaces of revolution), Conjecture 1 holds for the subclass of surfaces that are embedded and contained in a halfspace. It is therefore of interest to obtain natural geometric conditions that force a compact embedded constant mean curvature surface to be contained in a halfspace.

One result of our paper is to give the following sufficient condition for a compact constant mean curvature surface to be contained in a halfspace.