

Comparison Principle and Convexity Preserving Properties for Singular Degenerate Parabolic Equations on Unbounded Domains

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ABSTRACT. We prove comparison theorems for viscosity solutions of singular degenerate parabolic equations of general form in a domain not necessarily bounded. We also prove that the concavity of solutions is preserved as time develops under additional assumptions on the equations. Both results apply to various equations including the mean curvature flow equation where every level set of solutions moves by its mean curvature.

1. Introduction. This paper is concerned with viscosity solutions of (possibly) degenerate parabolic equations of the form:

$$(1.1) \quad u_t + F(\nabla u, \nabla^2 u) = 0 \quad \text{in } Q = (0, T] \times \Omega,$$

or more general equations

$$(1.2) \quad u_t + F(t, x, u, \nabla u, \nabla^2 u) = 0 \quad \text{in } Q = (0, T] \times \Omega,$$

where Ω is a domain in \mathbf{R}^n and $T > 0$. The equations are allowed to be singular in the sense that F has a singularity at $\nabla u = 0$. The unknown u will always be a real-valued function on Q ; $\partial_t u$, ∇u and $\nabla^2 u$ denote respectively the time derivative of u , the gradient of u and the Hessian of u in space variables. As explained in [1, 6] a certain class of singular degenerate parabolic equations is important to study motion of a hypersurface whose speed locally depends on the normal and its derivatives. A typical example is

$$(1.3) \quad u_t - |\nabla u| \operatorname{div} \left(\frac{\nabla u}{|\nabla u|} \right) = 0$$