

# *The Lifespan of Smooth Solutions to the Three-Dimensional Compressible Euler Equations and the Incompressible Limit*

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**0. Introduction.** In this paper, we will obtain a lower bound for the lifespan of classical solutions to the three-dimensional compressible isentropic Euler equations

$$(0.1) \quad \partial_t \rho + \nabla \cdot \rho u = 0$$

$$(0.2) \quad \rho(\partial_t u + u \cdot \nabla u) + \nabla \rho^\gamma = 0,$$

with initial data of the form

$$(0.3) \quad \rho(0, x) = \bar{\rho} + \varepsilon \rho_0^\varepsilon(x) > 0, \quad u(0, x) = \varepsilon u_0^\varepsilon(x),$$

representing a perturbation of order  $\varepsilon$  from the constant background state  $(\rho, u) = (\bar{\rho}, 0)$ . For cosmetic reasons, we will omit the dependence of the solution on the parameter  $\varepsilon$ . Here,  $\bar{\rho}$  is a positive constant,  $\gamma > 1$ , and the functions  $\rho_0^\varepsilon$  and  $u_0^\varepsilon$  are uniformly bounded in the Schwartz space  $\mathcal{S}(\mathbb{R}^3)$ , for all  $\varepsilon > 0$ . We shall show that if  $u_0^\varepsilon$  is *irrotational*, then the lifespan  $T_\varepsilon$  of the classical solution of (0.1), (0.2), (0.3) satisfies the lower bound

$$(0.4) \quad T_\varepsilon > \exp(C/\varepsilon)$$

for small  $\varepsilon$ , extending the generic lower bound

$$(0.5) \quad T_\varepsilon > C/\varepsilon$$

which holds quite generally for symmetric hyperbolic systems in any number of space dimensions.

Our interest in the question is two-fold. First, it was shown by the author [10] that the life span is bounded above by  $T_\varepsilon < \exp(C/\varepsilon^2)$  under mild conditions