

Representation and Refinement for Reflexive Operator Algebras with Completely Distributive Commutative Subspace Lattice

J. L. ORR & S. C. POWER

ABSTRACT. An intrinsic spectral representation is obtained for commutative projection lattices (on Hilbert space) which are completely distributive. This representation is based on the partially ordered set of meet-irreducible projections of the lattice, and it is used to obtain a refinement theory for multiple tensor products of reflexive operator algebras with such lattices as invariant projection lattices.

1. Introduction. A completely distributive CSL algebra, or CDCSL algebra, is a reflexive operator algebra on a complex Hilbert space which has a commutative invariant subspace projection lattice and which is the weakly closed linear span of its rank one elements. The latter property is equivalent to the complete distributivity of the lattice, which explains the terminology. We always assume that the underlying Hilbert space is separable. The class of CDCSL algebras is closed under orthogonal direct sums and spatial tensor products and in many ways is the most tractable class of CSL algebras. In this paper it will be shown that the class has the finite refinement property in the following sense; if $\mathcal{A}_1, \dots, \mathcal{A}_r$ and $\mathcal{B}_1, \dots, \mathcal{B}_s$ are irreducible CDCSL algebras such that the spatial tensor products $\mathcal{A}_1 \otimes \dots \otimes \mathcal{A}_r$ and $\mathcal{B}_1 \otimes \dots \otimes \mathcal{B}_s$ are unitarily equivalent, and if none of the factor algebras is one-dimensional or has nontrivial uniform multiplicity, then there exist CDCSL algebras $\mathcal{C}_{ij}, 1 \leq i \leq r, 1 \leq j \leq s$, such that

$$\begin{aligned} \mathcal{A}_i &= \mathcal{C}_{i1} \otimes \dots \otimes \mathcal{C}_{is}, & 1 \leq i \leq r, \\ \mathcal{B}_j &= \mathcal{C}_{1j} \otimes \dots \otimes \mathcal{C}_{rj}, & 1 \leq j \leq s. \end{aligned}$$

The terminology 'finite refinement property' is borrowed from the refinement theory of connected antisymmetric reflexive binary relations [CJT] and this theory lies at the heart of our proof. The multiplicity requirement here is that