

# *Isometries of $L_p(X;L_q)$ and Equimeasurability*

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**Introduction.** Let  $(X, \mathcal{A}, \sigma)$  and  $(X', \mathcal{A}', \sigma')$  be finite measure spaces, and  $p \geq 1$ . Given a Banach space  $E$ , denote by  $I(E)$  the set of all linear isometries from  $E$  to itself. Let  $L_p(X; E)$  be the Lebesgue-Bochner space of (equivalence classes of) strongly measurable functions  $f : X \rightarrow E$  with

$$\|f\| = \left( \int_X \|f(x)\|^p d\sigma(x) \right)^{1/p} < \infty.$$

We are going to characterize the isometries of Lebesgue-Bochner spaces. Let  $U : X' \rightarrow I(E)$  be a strongly measurable mapping (i.e., for each  $e \in E$ , the function  $\|U(x')e\|$  is  $\sigma'$ -measurable),  $\Phi$  be a rearrangement operator induced by a regular set isomorphism  $\varphi : \mathcal{A} \rightarrow \mathcal{A}'$ , and  $h : X' \rightarrow \mathbf{R}$  be a measurable function satisfying  $\sigma(A) = \int_{\varphi(A)} |h(x')|^p d\sigma'(x')$  for all sets  $A \in \mathcal{A}$ . Then the operator  $T : L_p(X; E) \rightarrow L_p(X'; E)$  defined by

$$(1) \quad Tf(x') = h(x')U(x')(\Phi f(x'))$$

is a linear isometry.

In the classical papers of S. Banach [3] and J. Lamperti [33], the scalar case ( $E = \mathbf{R}$  or  $E = \mathbf{C}$ ) was treated. If  $p \neq 2$ , then all isometries of  $L_p$ -spaces of scalar functions have the form (1) (without the mapping  $U$ ). Moreover, isometries of subspaces of  $L_p$  are of the same form if  $p \neq 2, 4, 6, \dots$ . The following result sets the stage for the recent progress in the investigation of the isometries of subspaces of  $L_p$ :

**Equimeasurability Theorem.** *Let  $p > 0$ ,  $p \neq 2, 4, 6, \dots$  and let  $H$  be a subspace of  $L_p(X, \sigma)$  containing the function  $\mathbf{1}(x) \equiv 1$ . If  $T : H \rightarrow L_p(X', \sigma')$  is a linear isometry with  $T\mathbf{1} = \mathbf{1}$ , then every function  $f \in H$  and its image  $Tf$  are equimeasurable with respect to  $\sigma$  and  $\sigma'$ , respectively (i.e.,  $\sigma\{x \in X : f(x) \in B\} = \sigma'\{x' \in X' : Tf(x') \in B\}$  for all Borel sets  $B$  in  $\mathbf{R}$ ).*