

Moore Cohomology, Principal Bundles, and Actions of Groups on C^ -algebras*

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Introduction. In recent years both topological and algebraic invariants have been associated to group actions on C^* -algebras. Principal bundles have been used to describe the topological structure of the spectrum of crossed products [18, 19], and as a result we now know that crossed products of even the very nicest non-commutative algebras can be substantially more complicated than those of commutative algebras [19, 5]. The algebraic approach involves group cohomological invariants, and exploits the associated machinery to classify group actions on C^* -algebras; this originated in [2], and has been particularly successful for actions of \mathbf{R} and tori ([19; Section 4], [21]). Here we shall look in detail at the relationship between these topological and algebraic invariants, with a view to analyzing the structure of the systems studied in [19; Section 2, 3].

Our starting point is a theorem of Rosenberg [21, Theorem 2.5] concerning the locally unitary actions of [18]. If A has Hausdorff spectrum X , and $\alpha : G \rightarrow \text{Aut}(A)$ is an action of an abelian group which is locally implemented by homomorphisms $u : G \rightarrow \mathcal{UM}(A)$, then the spectrum of the crossed product $A \rtimes_{\alpha} G$ is a principal \hat{G} -bundle over X ; the class $\zeta(\alpha)$ of the bundle determines α up to exterior equivalence, and all such bundles arise [18]. If G is connected, the range of α will often lie in the subgroup $\text{Inn}(A)$ of inner automorphisms, and then there is a class $c(\alpha)$ in the Moore cohomology group $H^2(G, C(X, \mathbf{T}))$ which is trivial when evaluated at points of X , and which vanishes precisely when α is implemented by a unitary group u [19, Section 0]. Rosenberg showed how to construct a principal bundle directly from a pointwise trivial class in $H^2(G, C(X, \mathbf{T}))$, and that his construction connects up the invariants $c(\alpha)$ and $\zeta(\alpha)$.

We aim to prove a version of Rosenberg's theorem for actions $\alpha : G \rightarrow \text{Aut}(A)$ which are locally unitary on a subgroup N . It was shown in [19; Section 2] that, provided $X = \hat{A}$ is a principal G/N -bundle, there is a commutative