

Fatou-Bieberbach Domains Arising from Polynomial Automorphisms

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An open subset $D \subset \mathbf{C}^2$ is known as a *Fatou-Bieberbach domain* if D is biholomorphically equivalent to \mathbf{C}^2 , but $D \neq \mathbf{C}^2$. The usual method of construction of such domains is as follows. Let f be a holomorphic diffeomorphism of \mathbf{C}^2 with an attracting fixed point p . Let $D(p)$ be the basin of attraction of p i.e.

$$(1) \quad D(p) = \{q \in \mathbf{C}^2 : \lim_{n \rightarrow \infty} f^n(q) = p\}.$$

If $D(p)$ is not all of \mathbf{C}^2 then $D(p)$ is a Fatou-Bieberbach domain (cf. [BM]). The holomorphic diffeomorphisms used in the original constructions of Fatou and Bieberbach were polynomial automorphisms of \mathbf{C}^2 . We will refer to basins of attraction for polynomial automorphisms as polynomial basins. Note that if f is an automorphism with an attracting cycle then any component of the basin of attraction of the attracting cycle is a polynomial basin.

We will consider the question of how these domains intersect one dimensional algebraic varieties. We might view these questions as an attempt to find a two dimensional analogue to the fact that a domain in \mathbf{C} equivalent to \mathbf{C} must be all of \mathbf{C} , in other words that such a domain must intersect every codimension one variety of \mathbf{C} . As remarked in [RR], the result of [G] shows that a Fatou-Bieberbach domain must intersect any set of three complex lines. Dektyarev points out in [D, p. 31], that a Fatou-Bieberbach domain must intersect every pair of disjoint lines, and he asks if it must intersect every complex line. This question is answered in [RR] where they construct an example of a Fatou-Bieberbach domain disjoint from a complex line. Their counterexample is constructed as a basin of attraction of a holomorphic diffeomorphism of \mathbf{C}^2 but the diffeomorphism is not a polynomial automorphism. If we restrict our attention to polynomial basins then the following theorem shows that Dektyarev's question has a positive answer.

Theorem 1. *Let $D(p)$ be a polynomial basin. Then $D(p)$ intersects every algebraic curve V . Furthermore $\overline{D(p)} \cap V$ is compact.*