

Small Amplitude Solutions of the Maxwell-Dirac Equations

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ABSTRACT. We prove the existence of a global solution of the Cauchy problem for the non-linear Maxwell-Dirac system in $3 + 1$ -dimensional space-time in Lorentz gauge. The space of initial data consists of functions sufficiently small with respect to suitable Sobolev weighted norms and satisfying the well-known constraint conditions. The essential novelty in the definition of the space of initial data is its explicit form and the possibility to choose initial data with noncompact support. The proof is based on the application of a gauge transformation leading to suitable gauge condition playing the role of the null condition of Klainerman and the “compatible form” condition of Bachelot.

1. Introduction. In this work we study the existence of a global solution of the classical Maxwell-Dirac non-linear system in four-dimensional space-time provided the initial data are sufficiently small with respect to suitable Sobolev norms.

The Maxwell-Dirac system is one of the key problems in quantum electrodynamics. The system describes the interaction between spinor and electromagnetic fields. For this, the Maxwell-Dirac system has been studied by many authors during the last three decades. L. Gross [15] was the first who proved in 1966 the uniqueness and local existence in time of solutions in suitable functional spaces. Later J. Chadam [4], J. Chadam and R. Glassey [5] considered the cases of $1 + 1$ - and $2 + 1$ -dimensional space-time. In [5], the case of $3 + 1$ -dimensional space-time is considered under the assumption that the magnetic field is zero. Another approach was developed by Y. Choquet-Bruhat [6] for the case of spinor field with zero mass. The approach is based on conformal invariance techniques of Penrose. The case of the Maxwell-Dirac equations in $3 + 1$ -dimensional Minkowski space is considered by M. Flato, J. Simon, and E. Taflin [8]. They constructed the modified Möller wave operator (Cauchy problem at $t = \infty$) and obtained a whole family of solutions on \mathbb{R}_t^+ . More than 10 years