

Julia Operators and Complementation in Kreĭn Spaces

MICHAEL A. DRITSCHEL & JAMES ROVNYAK

ABSTRACT. Operator methods are used to derive and in some ways extend the theory of complementation in Kreĭn spaces, especially in the direction of uniqueness questions. A close connection with the theory of Julia operators is established. An application is given to the characterization of the range of a Kreĭn space contraction operator.

Julia operators and complementation are familiar Hilbert space notions in invariant subspace theory and the study of contraction operators. Each has a Kreĭn space version that serves an additional purpose to circumvent difficulties in the indefinite setting, such as the failure of polar decompositions. The theory of complementation in Kreĭn spaces is due to de Branges [5] and is systematically applied in [6]. From a different direction, the authors [14] developed an abstract theory of Julia operators in Kreĭn spaces and used this to derive extension properties of contraction operators. As we shall see, the theories are essentially equivalent. In this paper, we use an elementary operator method to derive and in some ways extend the known complementation theory for Kreĭn spaces. We discuss uniqueness questions which arise in the indefinite setting. The result is a unified theory that has much the same completeness and flavor as in the Hilbert space case. Some of these results were announced in Appendix A in [14]. Julia operators and complementation have a long history, which we shall only give in part. Complementation is a geometric notion from invariant subspace theory as presented in de Branges and Rovnyak [7, 8]. In the Hilbert space case, M. Rosenblum gave seminar lectures on an operator approach to complementation in the 1960's, and similar methods are adopted in the survey of operator ranges by Fillmore and Williams [15]. The operator approach to complementation is used by other authors such as Sarason [19, 20] in applications in function theory. Schwartz [21] contains an elegant theory of operator ranges in both the Hilbert space and Kreĭn space settings and anticipates the ideas of complementation. The paper of Schwarz also makes a link with the very closely related point of