

Denseness of Numerical Radius Attaining Operators: Renorming and Embedding Results

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ABSTRACT. We prove that a strong geometrical condition (the unit ball is the balanced convex hull of a uniformly strongly exposed set), introduced by J. Lindenstrauss to deal with norm-attaining operators, is also sufficient for the denseness of numerical radius attaining operators. As a consequence, we show that every WCG Banach space can be equivalently renormed in such a way that the denseness of the numerical radius attaining operators is satisfied. We also obtain that every Banach space X is linearly isometric to a 1-complemented subspace of a Banach space Y with the same density character and such that the numerical radius attaining operators on Y are dense.

Introduction. In the following we will denote by X a Banach space, B_X and S_X will be the closed unit ball and the unit sphere of X , respectively, X^* the dual space of X and $L(X)$ the Banach space of (bounded and linear) operators on X .

We recall the notion of numerical range of an operator.

Definition 1. The *numerical range* of an operator $T \in L(X)$ is the set of scalars given by

$$V(T) = \{f(T(x)) : (x, f) \in \Pi(X)\},$$

where

$$\Pi(X) = \{(x, f) \in X \times X^* : \|x\| = \|f\| = f(x) = 1\}.$$

The *numerical radius* of T , $v(T)$ is

$$v(T) = \sup\{|\lambda| : \lambda \in V(T)\}.$$