

Behavior of Holomorphic Functions near Weakly Pseudoconvex Boundary Points

FRANK BEATROUS

ABSTRACT. L^p , uniform, and Lipschitz estimates are obtained for holomorphic Sobolev functions along parts of the boundary of a smooth domain, taking into account in a precise way the shape of the boundary. Flatness of the boundary is measured by its type, in the sense of Kohn, and estimates along subsets of the boundary depend on a lower bound for the type along the set.

Introduction. In this paper, we investigate the boundary behavior of holomorphic functions which, together with their derivatives up to some specified order, belong to a Bergman or Hardy space. We will obtain maximal L^p estimates, uniform estimates, and Lipschitz estimates which take into account in a precise way the flatness of parts of the boundary.

To indicate the nature of our results, we present a simple minded example which illustrates the sensitivity of the boundary estimates to the shape of the boundary. For $m \geq 2$, let

$$D_m = \{(z, w) \in \mathbb{C}^2 : |z|^2 + |w|^m < 1\}.$$

Thus, ∂D_m is strictly pseudoconvex except along the circle $E = \{(z, 0) : |z| = 1\}$, and ∂D_m becomes flatter along E as m increases. For $0 < p < \infty$, let A_s^p be the holomorphic subspace of the Sobolev space $W_s^p(D_m)$. (Thus, when s is a non-negative integer, A_s^p is the space of holomorphic functions on D_m having derivatives up to order s in $L^p(D_m)$.) For any holomorphic function f on D_m , it is an elementary consequence of Fubini's Theorem and the subharmonicity of the functions $w \mapsto |f(z, w)|^p$ that,

$$\int_{|z|<1} |f(z, 0)|^p (1 - |z|^2)^{2/m} dA(z) \leq \frac{1}{2\pi} \int_{D_m} |f|^p dV,$$