

A Disc in the Ball Whose End Is an Arc

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ABSTRACT. In this paper we construct an arc in the boundary of the unit ball in C^6 which bounds an analytic disc.

Denote by Δ the open unit disc in C and by B_N the open unit ball in C_N . We prove the following:

Theorem. *There are an arc $L \subset bB_6$ and a continuous map $f : \bar{\Delta} \rightarrow \bar{B}_6$ such that*

- (i) *f is holomorphic, regular and one to one on Δ ;*
- (ii) *$f(b\Delta) = L$.*

Thus $D = f(\Delta)$ is a disc holomorphically embedded into B_6 whose end $\bar{D} \setminus D$ is L . The ends of discs were studied in [2, 3]. From [3, Th.II.1] it follows that the arc L above has positive two dimensional Hausdorff measure. L above is an arc in the sphere which is not polynomially convex. It is known that there are arcs in C^N which are not polynomially convex [6, 9]; that such an arc can be found in the sphere seems to be new.

PROOF

▷ *Part 1.* Let $J \subset C$ be a closed arc such that the algebra A_J of continuous functions on the Riemann sphere S which are holomorphic on $S \setminus J$ contains nonconstant functions [8]; let F_1 be such a function. Choose $a, b \in C$ so that $F_1(a) \neq F_1(b)$ and define $F_2(\zeta) = (F_1(\zeta) - F_1(a))/(\zeta - a)$, $F_3(\zeta) = (F_1(\zeta) - F_1(b))/(\zeta - b)$. Then $F_2, F_3 \in A_J$ and the map $\zeta \mapsto \Phi(\zeta) = (F_1(\zeta), F_2(\zeta), F_3(\zeta))$ is one to one on S so $\Lambda = \Phi(J)$ is an arc. This construction of an arc Λ in C^3 which is not polynomially convex is due to J. Wermer [8].

Replacing F_1 at the beginning by $\zeta \mapsto \zeta^k (F_1(\zeta) - F_1(\infty))$ for a suitable k we may assume that the derivative of $\xi \mapsto F_1(\xi^{-1})$ at 0 is different from 0. Further, we may choose a, b so that $F_1'(a) \neq 0, F_1'(b) \neq 0$. It is then easy to see that the map Φ is regular on $S \setminus J$.