

Regularity of Solutions to Some Degenerate Double Obstacle Problems

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ABSTRACT. We study double obstacle problems for a class of elliptic operators modelled on the p -Laplacian operator: $Qu = \operatorname{div}(|Du|_{p-2}Du)$ for $p > 1$. When the obstacles are $C^{1,\alpha}$ (with α sufficiently small), so is the solution; this regularity is proved both locally and up to the boundary.

Introduction. Recently Dal Maso, Mosco, and Vivaldi [5] and Kilpelainen and Ziemer [12] have studied solutions of elliptic variational inequalities with constraints of the form $\psi_1 \leq u \leq \psi_2$ for given functions ψ_1 and ψ_2 . If the obstacles satisfy suitable Wiener-type conditions, then the solutions are shown to be continuous and a modulus of continuity for the ψ 's leads to a corresponding one for u ; in particular, if the ψ 's are Hölder continuous, so is u (with a possibly different exponent). Because the hypotheses in these works include linear elliptic operators with bounded measurable coefficients (Kilpelainen and Ziemer also study nonlinear problems), these results are best possible.

Here we show that additional hypotheses on the operator lead to Hölder regularity of the gradient of u if the ψ 's are smooth enough. Our approach is based on potential theoretic ideas of Caffarelli and Kinderlehrer [1] with some modifications from work of the author [17], which is based on Giaquinta's approach to the one-obstacle problem [7]. An alternative approach based on Choe's proof [2] of higher regularity for the one-obstacle problem appears in [3] and one based on Mu's proof [21] of this result is in [22], but other methods [4], [18], [23] seem not to be applicable to the double-obstacle problem.

Our hypotheses on the associated differential operator, which include the p -Laplacian operator, are the same as in [16], [17]: Let $g \in C^1(0, \infty)$ and suppose there are positive conditions $\delta \leq g_0$ such that

$$(0.1) \quad \delta \leq \frac{tg'(t)}{g(t)} \leq g_0 \quad \text{for } t > 0.$$

For Ω an open subset of \mathbf{R}^n , we introduce a vector function A and a scalar function B , both defined on $\Omega \times \mathbf{R} \times \mathbf{R}^n$, and we assume there are nonnegative