

Single Point Blow-Up for N-Dimensional Quasilinear Equations with Gradient Diffusion and Source

VICTOR A. GALAKTIONOV & SERGEY A. POSASHKOV

ABSTRACT. In this paper we consider the Cauchy problem for the quasilinear parabolic equation with source

$$(*) \quad u_t = A_0(u) \equiv \nabla \cdot (|\nabla u|^\sigma \nabla u) + u^\beta \quad \text{for } x \in \mathbf{R}^N, t > 0,$$

where $\sigma > 0$ and $\beta > \sigma + 1$ are fixed constants. We study behavior of the solution $u = u(|x|, t) \geq 0$ to the equation $(*)$ existing on a finite time interval $(0, T_0)$ such that $\overline{\lim} u(0, t) = +\infty$ as $t \rightarrow T_0^- < +\infty$. Upper and lower estimates of the final profile $u(x, T_0)$ near the single blow-up point are proved by different methods. These establish the sharp exponent of a power character of singularity near the origin: $u(x, T_0) \cong \text{const } |x|^{-\gamma}$, where $\gamma = \frac{\sigma+2}{[\beta-(\sigma+1)]}$. Some generalizations are also given.

1. Introduction: Main Results. In this paper we consider the Cauchy problem for a quasilinear degenerate parabolic equation with gradient diffusion term and source

$$(1.1) \quad u_t = A_0(u) \equiv \nabla \cdot (|\nabla u|^\sigma \nabla u) + u^\beta \quad \text{for } x \in \mathbf{R}^N, t > 0,$$

$$(1.2) \quad u(x, 0) = u_0(x) \geq 0 \text{ in } \mathbf{R}^N; u_0 \in C^1(\mathbf{R}^N), \sup u_0 < \infty,$$

where $\sigma > 0$ and $\beta > 1$ are fixed constants, $N \geq 1$.

It is well-known (see e.g., [8], [20], [24], [31]) that under some hypotheses Problem (1.1), (1.2) has no global in time solution, i.e., the weak solution $u = u(x, t) \geq 0$ exists in a finite time interval $(0, T_0)$ and it blows up as $t \rightarrow T_0$:

$$(1.3) \quad \overline{\lim}_{t \rightarrow T_0^-} \sup_{x \in \mathbf{R}^N} u(x, t) = +\infty.$$