

On the Positive Solutions of Semilinear Equations $\Delta u + \lambda u + hu^p = 0$ on Compact Manifolds. Part II

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ABSTRACT. In this paper, we study the existence and non-existence of the positive solutions of the semilinear equation $\Delta u + \lambda u + hu^p = 0$ on compact Riemannian manifolds, where $\lambda > 0$, $p > 1$ and $h(x)$ satisfies assumptions (A-1) and (A-2) below. We prove that there exists a constant λ_0 , $0 < \lambda_0 < \infty$ such that for any $0 < \lambda < \lambda_0$, there exist two distinct solutions $u_1(\lambda, x) < u_2(\lambda, x)$; for $\lambda = \lambda_0$, there exists a unique solution $u(\lambda_0, x)$; for $\lambda > \lambda_0$, there is no positive solution of the equation above.

1. Introduction. This paper is a continuation of our previous paper [10], where we studied the existence, non-existence and uniqueness of the positive solutions of the semilinear equation

$$\Delta u + \lambda u - hu^p = 0$$

on the compact Riemannian manifolds (M, g) , where $p > 1$, $h(x) \geq 0$ on M .

Now let (M, g) be a Riemannian manifold of dimension ≥ 3 with scalar curvature k and let K be a given function on M . One may ask the question: can we find a new metric g^* on M such that K is the scalar curvature of g^* and g^* is pointwise conformal to g (i.e., $g^* = u^{4/(n-2)}g$ for some $u > 0$ on M)? This is equivalent to the problem of finding positive solutions of the equation

$$\frac{4(n-1)}{n-2} \Delta u - ku + Ku^{(n+2)/(n-2)} = 0,$$

where Δ is the Laplace-Beltrami operator (simply Laplacian) in the g metric (see e.g., [3], Chapter 6 and [9]).