

Stability of Weak Shocks in $\lambda - \omega$ Systems

TODD M. KAPITULA

ABSTRACT. We consider the $\lambda - \omega$ reaction diffusion equation. Howard and Kopell ([HK]) show that these equations support the existence of “weak shocks,” which are travelling wave solutions that join plane waves having sufficiently close wave numbers. Furthermore, they show that the wave satisfies an entropy condition at the end states. In studying the stability of these solutions we are faced with the difficulty that in an unweighted space the essential spectrum contains $\gamma = 0$. However, in an exponentially weighted L^∞ space the spectrum is contained in the left-half-plane. We show that these solutions are stable in a polynomially weighted L^∞ space. Furthermore, we show that the rate at which the perturbation decays as $t \rightarrow \infty$ depends on the growth rate of the polynomial near infinity.

1. Introduction. Consider the $\lambda - \omega$ reaction-diffusion equation

$$(1.1) \quad \begin{pmatrix} u \\ v \end{pmatrix}_t = \begin{pmatrix} u \\ v \end{pmatrix}_{xx} + \begin{pmatrix} \lambda(r) & -\omega(r) \\ \omega(r) & \lambda(r) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

where $(x, t) \in \mathbf{R} \times \mathbf{R}^+$, $r^2 = u^2 + v^2$, and λ and ω are C^2 functions of r . Kopell and Howard ([KH]) first consider the existence problem of “plane wave” solutions for reaction-diffusion equations. Plane waves are solutions which are 2π -periodic in their arguments. They show that under certain conditions on the diffusivity matrix and the nonlinearity these solutions arise as a Hopf bifurcation. Equation (1.1) exhibits all the fundamental properties of their general theory, and yet is amenable to explicit calculations.

Howard and Kopell ([HK]) consider (1.1) as a mathematical model for the Belousov-Zhabotinskii reaction ([Be], [Zh]). In [HK] a mathematical understanding is given for the interaction of “target patterns,” which are concentric outwardly propagating rings. In any given radial direction, away from the center