

Tangential Characterizations of Hardy–Sobolev Spaces

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1. Introduction. Let Ω be a bounded strictly pseudoconvex domain in C^n with C^∞ boundary $\partial\Omega$. Thus, we suppose that there is a C^∞ defining function $r : C^n \rightarrow R$, a neighborhood \mathcal{O} of $\partial\Omega$ and a constant $C > 0$ such that

$$\Omega = \{\zeta : r(\zeta) < 0\},$$

$$\partial\Omega = \{\zeta : r(\zeta) = 0\},$$

$$|\nabla r| \neq 0,$$

everywhere on \mathcal{O} , and that the Levi form of r is positive definite on \mathcal{O} :

$$\sum_{j,k} \frac{\partial^2 r}{\partial \zeta_j \partial \bar{\zeta}_k}(\zeta) w_j \bar{w}_k \geq C|w|^2$$

for all $\zeta \in \mathcal{O}$ and all $w \in C^n$.

If F is a holomorphic function on Ω , we will say that F belongs to the Hardy space $H^p(\Omega)$ if

$$\|F\|_p^p = \sup_{\varepsilon < 0} \int_{\partial\Omega_\varepsilon} |F(\zeta)|^p d\sigma < \infty,$$

where $d\sigma$ denotes the volume element (or “surface area measure”) on the manifold

$$\partial\Omega_\varepsilon = \{\zeta : r(\zeta) = \varepsilon\}.$$

Similarly, we say that F belongs to the Hardy–Sobolev space $H_k^p(\Omega)$ if all derivatives of F of order less than or equal to k belong to $H^p(\Omega)$. It will be convenient to use multi-index notation: if $I = (k_1, k_2, \dots, k_n)$, where each k_i is a non-negative integer, then

$$D_I = \frac{\partial^{|I|}}{\partial \zeta_1^{k_1} \dots \partial \zeta_n^{k_n}}$$

and

$$\bar{D}_I = \frac{\partial^{|I|}}{\partial \bar{\zeta}_1^{k_1} \dots \partial \bar{\zeta}_n^{k_n}},$$