

# *Some Counterexamples in Subalgebras of $L^\infty(D)$*

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ABSTRACT. Let  $\mathbb{D}$  denote the open unit disc in the complex plane and let  $dA$  denote the usual two-dimensional Lebesgue area measure. In this paper, we think of  $H^\infty(\mathbb{D})$  as a closed subalgebra of  $L^\infty(\mathbb{D}, dA)$ . Motivated by the Chang–Marshall theorem for subalgebras of  $L^\infty$  on the circle containing  $H^\infty$ , we study the problem of when a subalgebra of  $L^\infty(\mathbb{D}, dA)$  of the form  $H^\infty(\mathbb{D})[\bar{f}]$ , where  $f$  is a function in  $H^\infty$ , is generated by  $H^\infty$  and the complex conjugates of the interpolating Blaschke products in  $H^\infty(\mathbb{D})[\bar{f}]$ .

**1. Introduction.** Let  $\mathbb{D}$  denote the open unit disc in the complex plane and let  $dA$  denote the usual two-dimensional Lebesgue area measure. We let  $H^\infty(\mathbb{D})$  denote the algebra of bounded analytic functions on  $\mathbb{D}$ . Unless otherwise stated, we will be thinking of  $H^\infty(\mathbb{D})$  as a closed subalgebra of  $L^\infty(\mathbb{D}, dA)$ . For a function  $u \in L^\infty(\mathbb{D}, dA)$ , we denote the closed subalgebra generated by  $H^\infty(\mathbb{D})$  and  $u$  by  $H^\infty(\mathbb{D})[u]$ .

A Blaschke product  $b$  with zero sequence  $\{z_n\}$  satisfying

$$\inf_n (1 - |z_n|^2) |b'(z_n)| = \delta(b) > 0$$

is said to be an interpolating Blaschke product. If, in addition, the zero sequence satisfies

$$\liminf_{n \rightarrow \infty} (1 - |z_n|^2) |b'(z_n)| = 1,$$

the Blaschke product is called a thin Blaschke product and the sequence is a thin sequence.

When all the algebras above are considered on the unit circle, a great deal is known about them. Denoting the algebra of essentially bounded measurable functions on the unit circle by  $L^\infty(\mathbb{T})$ , the algebra  $H^\infty$  may be thought of as a closed subalgebra of  $L^\infty(\mathbb{T})$  by taking radial limits of the function in  $H^\infty$ .