

On the Barenblatt Equation of Elasto-Plastic Filtration

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ABSTRACT. In this paper we consider the equation

$$(*) \quad u_t + \gamma|u_t| = \Delta u, \quad 0 < |\gamma| < 1,$$

which was first studied by Barenblatt and his collaborators in the context of flow of elastic fluids in elasto-plastic porous media. We establish the existence and uniqueness of a singular self-similar solution $B(x, t) = t^{-\alpha/2} f(\eta)$, in which the decay exponent α is uniquely determined as a function of γ (and N , the space dimension). Such α is called *the anomalous exponent*. Qualitative properties of f and α are investigated. In particular, it is shown that α is a strictly increasing function of γ with limit values $\alpha(-1) = \max\{N - 2, 0\}$ and $\alpha(1) = \infty$.

The importance of this self-similar solution lies in the fact that it represents (up to a constant) the large-time behaviour of all solutions of $(*)$ with nonnegative initial data which decay sufficiently fast as $|x| \rightarrow \infty$, that is $u \approx cB$ for t large enough. Thus, the anomalous exponent describes the decay rate of all such solutions. The existence and properties of the asymptotic constant c are also investigated.

Introduction. We consider the equation

$$(E) \quad u_t + \gamma|u_t| = \Delta u, \quad 0 < |\gamma| < 1$$

in $Q = \{(x, t) : x \in \mathbf{R}^N, t > 0\}$. This equation arises in the theory of an elastic fluid in an elasto-plastic porous medium, under the assumption that the porous medium is irreversibly deformable, and was studied by Barenblatt and his collaborators, see [B], [BK], or the recent book [BER]. There the equation is written in the form

$$(E') \quad F(u_t) = \Delta u \text{ with } F(s) = \begin{cases} (1 + \gamma)s & \text{for } s \geq 0 \\ (1 - \gamma)s & \text{for } s < 0. \end{cases}$$