

# Analytic Continuation in Bergman Spaces and the Compression of Certain Toeplitz Operators

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ABSTRACT. Let  $G$  be a Jordan domain and  $K \subset G$  be relatively closed with  $\text{Area}(K) = 0$ . Let  $A^2(G \setminus K)$  and  $A^2(G)$  be the Bergman spaces on  $G \setminus K$ , respectively  $G$  and define  $\mathcal{N} = A^2(G \setminus K) \ominus A^2(G)$ . In this paper we show that with a mild restriction on  $K$ , every function in  $\mathcal{N}$  has an analytic continuation across the analytic arcs of  $\partial G$  that do not intersect  $K$ . This result will be used to discuss the Fredholm theory of the operator  $C_f = P_{\mathcal{N}} T_f|_{\mathcal{N}}$ , where  $f \in C(\bar{G})$  and  $T_f$  is the Toeplitz operator on  $A^2(G \setminus K)$ .

**1. Introduction.** Let  $U$  be a bounded, open, connected, non-empty subset of  $\mathbf{C}$ . Let  $L^2(U)$  denote the Hilbert space of complex valued measurable functions (with respect to two-dimensional Lebesgue measure) on  $U$  which are square integrable. The inner product is given by  $\langle f, g \rangle = \int_U f \bar{g} \, dA$  and the norm of a function  $h \in L^2(U)$  will be given by  $\|h\|_2 = \langle h, h \rangle^{1/2}$ . The *Bergman space*, denoted  $A^2(U)$ , is the closed subspace of all functions in  $L^2(U)$  which are analytic on  $U$ . We motivate this paper with the following example. Let  $D$  be the unit disk and  $K \subset D$ ,  $K$  compact with  $\text{Area}(K) = 0$ . If  $f \in A^2(D \setminus K)$ , then in some annulus  $A$  contained in  $D$ ,  $f$  has a Laurent series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n z^n.$$

Define  $f_1$  and  $f_2$  in  $A$  by

$$(1) \quad \begin{aligned} f_1(z) &= \sum_{n=0}^{\infty} c_n z^n \\ f_2(z) &= \sum_{n=-\infty}^{-1} c_n z^n \end{aligned}$$