

Weighted Norm Inequalities for Commutators of Strongly Singular Integrals

J. GARCÍA-CUERVA, E. HARBOURE,
C. SEGOVIA, & J. L. TORREA

ABSTRACT. We study L^p inequalities with different but related weights for commutators of a strongly singular integral and a multiplication operator. These commutators turn out to be controlled by commutators of fractional order of the Hardy-Littlewood maximal operator. The boundedness properties of these are obtained by extrapolation from infinity.

1. Notations and Definitions. We denote by R^n the n -dimensional euclidean space. The Lebesgue measure of a Lebesgue measurable set $E \subset R^n$ is denoted by $|E|$. If Q is a cube in R^n and γ is a real number, then γQ shall stand for the cube with the same center as Q and side γ times that of Q . A weight $w(x)$ is a non-negative measurable function on R^n . The measure associated with w is the set function given by $w(E) = \int_E w(x) dx$. By $L^p(w)$, $0 < p < \infty$, we denote the space of all Lebesgue measurable functions $f(x)$ such that

$$\|f\|_{L^p(w)} = \left(\int_{R^n} |f(x)|^p w(x) dx \right)^{1/p} < \infty.$$

The average of a locally integrable function f over a cube Q is defined as $m_Q f = |Q|^{-1} \int_Q f(x) dx$. Given a cube $Q \subset R^n$ and $0 < r < \infty$, the Hardy-Littlewood maximal function with respect to Q of a function $f(x)$, $x \in Q$, is defined as

$$M_r^Q(f)(x) = \sup_{x \in J \subset Q} \left(|J|^{-1} \int_J |f(y)|^r dy \right)^{1/r}$$

where J is any cube satisfying the conditions. If $Q = R^n$ we simply write $M_r(f)$ instead of $M_r^{R^n}(f)$ and if, in addition, $r = 1$, just $M(f)$. Given $1 < p < \infty$, a