

3-Manifolds with Geometric Structure and Approximate Fibrations

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ABSTRACT. Let $p : M \rightarrow B$ be a proper map defined on an orientable 5-manifold M such that each $p^{-1}b$ is homeomorphic to a fixed closed, orientable 3-manifold N . The purpose of this paper is to determine those geometric 3-manifolds N for which p is invariably an approximate fibration; more explicitly, it is to see whether the presence of a particular geometric structure on N causes p to be one. Many of the manifolds N with the structure of S^3 are known to have this approximate fibration-inducing feature; the majority of those with E^3 and with $H^2 \times R$ structures do not, nor do the two with $S^2 \times R$ structure. New results include: (1) all hyperbolic, Sol, and $SL_2(R)$ 3-manifolds induce approximate fibrations in this setting; and (2) some, but not all, Nil manifolds that fiber over S^1 have the same feature, as do the other Nil manifolds that fail to fiber over S^1 .

Introduction. Viewed in widest perspective, this paper aims to provide an intrinsic, or natural, structure theorem for rapidly detecting certain useful mappings called approximate fibrations, which were first studied by Coram and Duvall [CD1]. Closely connected to the more familiar classical notion of fibrations, such mappings have the strongest regularity property possible in the setting at hand. A significant attraction is an associated long exact sequence, comparable to that for fibrations, relating homotopy groups of the domain, image and fiber [CD1, Lemma 3.5].

More specifically, the paper strives to identify closed n -manifolds N^n which automatically induce approximate fibrations, when the setting consists of a proper, surjective mapping $p : M \rightarrow B$ defined on an $(n+2)$ -manifold M such that each point preimage is a copy of N^n , up to shape. Since distinct point preimages of any approximate fibration between connected ANR's always have