

Interior Gradient Estimates for Solutions of Prescribed Curvature Equations of Parabolic Type

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ABSTRACT. We establish apriori interior gradient estimates for a class of nonlinear parabolic equations naturally related to the problem of the normal deformation of a hypersurface by a function of its curvatures.

1. Introduction. In this paper we establish *a priori* interior gradient estimates for fully nonlinear equations of the form

$$(1.1) \quad F[D^2u] \equiv f(\kappa) = \frac{1}{w} \psi(x, t, u, \nu, u_t), \quad w = (1 + |Du|^2)^{1/2}$$

where $\nu = (\frac{Du}{w}, -\frac{1}{w})$ denotes the downward directed unit normal on the spatial graph S of the function, $u, \kappa = (\kappa_1, \dots, \kappa_n)$ is the n -tuple of principal curvatures of S with respect to the upward unit normal vector, $\psi \in C^1(\mathbb{R}^n \times \mathbb{R}^+ \times \mathbb{R} \times S^n \times \mathbb{R})$ is a prescribed function, and f is a smooth symmetric function defined in an open convex symmetric cone $\Gamma \subset \mathbb{R}^n$, with vertex at the origin, and containing the positive cone $\Gamma^+ \equiv \{\kappa \in \mathbb{R}^n : \text{each component } \kappa_i > 0\}$. The equation is assumed to be parabolic, i.e. (see [1]),

$$(1.2) \quad f_i \equiv \frac{\partial f}{\partial \kappa_i} > 0 \text{ in } \Gamma, \quad \text{for } 1 \leq i \leq n,$$

and

$$(1.3) \quad \psi_\tau(x, u, t, \nu, \tau) > 0.$$

Equations of form (1.1) are naturally related to the problem of the normal deformation of a hypersurface by a function of its curvatures. The corresponding elliptic equations have first been studied by Caffarelli, Nirenberg and Spruck [1]–[3]. Following them, we call a function u *admissible* if, at every point on the graph $S = S_t$ of $u(\cdot, t)$, $\kappa \in \Gamma$, for any t .