

CR-transformations of Real Manifolds in \mathbf{C}^n

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1. Introduction. The notions of *CR*-manifold and *CR*-function are now fundamental in Several Complex Variables. The development of the theory of *CR*-manifolds and *CR*-functions naturally requires us to also introduce and to study *CR*-transformations.

Let M_1, M_2 be *CR*-manifolds in \mathbf{C}^n . A mapping $f = (f_1, \dots, f_n) : M_1 \rightarrow M_2$ is called a *CR*-mapping if all components f_j are *CR*-functions on M_1 , i.e., they satisfy the tangential Cauchy-Riemann conditions. Here the functions f_j are not necessarily differentiable because weak tangential Cauchy-Riemann conditions make sense for continuous functions and even for distributions. We restrict ourselves to continuous *CR*-mappings to make sense out of the expression $f : M_1 \rightarrow M_2$.

Definition 1. A mapping $f : M_1 \rightarrow M_2$ is called a *CR*-homeomorphism if

- (i) f is a homeomorphism,
- (ii) f is a *CR*-mapping,
- (iii) f^{-1} is also a *CR*-mapping.

Definition 2. A *CR*-manifold M in \mathbf{C}^n is called locally k -*CR*-straightened near a point $p^0 \in M$ if there exists a *CR*-manifold M_0 in \mathbf{C}^{n-k} and a *CR*-homeomorphism

$$f : (M_0 \times \mathbf{C}^k) \cap V \rightarrow M \cap U,$$

where $U, V \subset \mathbf{C}^n$ are some neighborhoods of the points $p^0, q^0 = f^{-1}(p^0)$ respectively.

In Section 2 we discuss the problem of *CR*-straightening. Each k -*CR*-straightened manifold M is foliated by complex varieties of dimension k . We shall assume everywhere below that these varieties are non-singular, i.e., they are complex manifolds in \mathbf{C}^n .

Each point p of a k -*CR*-straightened manifold m belongs to a unique leaf S of the foliation. We denote by π_p the tangent plane $T_p(S) \subset T_p^c(M)$ to S at