

# *The Necessity of the Wiener Test for Some Semi-linear Elliptic Equations*

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ABSTRACT. Regular boundary points for solutions to the Dirichlet problem for  $-\operatorname{div}(A(x)\nabla u) = f(x, u, \nabla u)$  in an arbitrary bounded domain in  $n$ -dimensional space, are regular points for Laplace's equation, when  $A(x)$  is Dini continuous and  $f$  has quadratic growth in  $\nabla u$ .

**1. Introduction.** The purpose of this note is to investigate the question of the necessity of the classical Wiener Test at a boundary point for the Dirichlet problem for the nonlinear equation

$$(1.1) \quad -\operatorname{div}(A(x)\nabla u) = f(x, u, \nabla u)$$

in an arbitrary bounded domain  $\Omega \subset \mathbf{R}^n$ ,  $n \geq 2$ . Here, the  $n \times n$  matrix  $A(x)$  is continuous, symmetric, and uniformly elliptic on  $\Omega$ ; the  $f$  is a given smooth function with at most quadratic growth in  $\nabla u =$  gradient of  $u$ ; see Section 2 below for the detailed definitions.

By a Wiener Test at a specific boundary point  $x_0 \in \partial\Omega$ , we mean a sharp geometric condition on the complement of  $\Omega$ ,  $\Omega^c$ , in the neighborhood of  $x_0$ , that insures that solutions to (1.1) will take on their preassigned continuous boundary data, continuously, at the point in question. Such a condition was first given in 1924 by N. Wiener in [W]. This condition can be restated as:  $x_0 \in \partial\Omega$  is a "regular point" in the above sense for the class of harmonic functions on  $\Omega$  iff

$$(1.2) \quad \int_0 r^{2-n} \operatorname{cap}(\Omega^c \cap B(x_0, r)) \frac{dr}{r} = +\infty.$$

Here  $B(x_0, r)$  is an open ball centered at  $x_0$  of radius  $r$  contained in  $\mathbf{R}^n$  and  $\operatorname{cap}(\cdot)$  is the classical Newtonian capacity set function defined for all Borel sets in  $\mathbf{R}^n$  when  $n \geq 3$ ; when  $n = 2$ ,  $\operatorname{cap}(\cdot)$  is the usual "conductor capacity" (see Section 3). The fact that (1.2) is also known to be both necessary and